ASSIGNMENT
For week 10/28-11/3

- Read the attached article on Math & Nature
- Find a news item that makes a mathematical claim of some sort
- Summarize the claim as you understand it
- Develop 2 (or more) knowledge questions that derive from that claim. Read the item on QUESTIONS if you need help in understanding what a Knowledge Question is.
- Share your findings with the class.

Mathematics and the Language of Nature

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Published in Mathematics and Sciences, edited by Ronald E. Mickens (Word Scientific, 1990)

1. Introduction

"God is a Mathematician", so said Sir James Jeans1. In a series of popular and influential books, written in the 1930s, the British astronomer and physicist suggested that the universe arises out of pure thought that is couched in the language of abstract mathematics. But why should God think only in mathematics? After all, some of most impressive achievements of the human race have involved architecture, poetry, drama and art. Could the essence of the universe not equally be captured in a symphony, or unfolded within a poem?

Three centuries earlier, Galileo had written, "Nature's great book is written in mathematical language" an opinion that has wholeheartedly been endorsed by physicists of our own time. Mathematics today occupies such an important position in physics that some commentators have argued that it has begun to lead and direct research in physics. In a frontier field , called Superstrings, some critics are arguing that mathematics is actually filling in the gaps left by the lack of any deep physical ideas. But why should mathematics play such a powerful role in physics? Is its central position inevitable? And is the present marriage between physics and mathematics always healthy, or are there ways in which mathematics may, at times, block creativity? In this essay I want to explore, in a speculative and free-wheeling way, some possible answers to these questions and to make some suggestions as to some radical developments in a language for the physical world.

2. The Role of Mathematics

While there are exceptions, it is generally true that great mathematics is studied for its own sake and without reference to anything outside itself. Mathematics has a beauty all its own and there is, for the mathematician, an aesthetic joy that comes from solving an important problem, no matter what value society may place on this activity. In this sense, mathematics has constantly sought to free itself from its practical origins.

Geometry, for example, began with rules for surveying, calculating the areas of fields and making astronomical studies and acts of navigation. Probability theory had its origins in the desire to raise gambling to a high art. But, very quickly, mathematics shook itself free from such pedestrian origins. While it is certainly true that some exceptional mathematicians have begun their studies with a concrete problem taken from the physical world, in the end, the mathematics they have developed has moved away from these specific cases in order to focus on more abstract relationships. Mathematics is not really concerned with specific cases but with the abstract relationships of thought that spring from these particular instances. Indeed, mathematics takes a further step of abstraction by investigating the relations between these relationships. In this fashion, the whole field moves away from its historical origins, towards greater abstraction and increasing beauty.

The English mathematician G. Hardy2 refused to justify mathematics in terms of its utility and pursued it as an art for its own sake. He seem to rejoice in the very abstraction of his own research and in its remoteness from practical applications. Indeed, Hardy once spoke of a monument so high that no one would ever be able to see the statue that was placed at its pinnacle - a fitting metaphor for his own, somewhat extreme, view of the role of mathematics.

In von Neumann words, mathematics is "the relation of relationships." Today it is possible to go further, for a that branch of mathematics called Category theory is not concerned with any particular field of mathematics but with the relationships between the different fields themselves! Mathematics at this level has the appearance of the purest and most rarefied thought. It is like a piece of music of such abstract perfection that the realization of a single performance would destroy its purity.

But it is exactly at this point that a staggering paradox hits us in the teeth. For abstract mathematics happens to work. It is useful. It is the tool that physicists employ in working with the nuts and bolts of the universe! Indeed, scientists of the old school referred to mathematics as "the handmaid of physics". But why should an abstract codification of pure thought, divorced from any reference to physical objects and material processes, be so useful in the daily practice of science? To echo Eugene Wigner's famous remark, mathematics is unreasonably effective.

There are many examples, from the history of science, of a branch of pure mathematics which, decades after its invention, suddenly finds a use in physics. There are also cases of a mathematical approach, developed for one specific purpose, that is later found to be exactly what is needed for some totally different area of physics.

Probability theory, first devised to deal with strategies of gambling, ends up as the exact language needed to give a molecular foundation to thermodynamics - the physical theory dealing with work and heat. But why should this be so? When Einstein formulated his general theory of relativity he discovered that the necessary mathematics had already been developed in the previous century. Similarly the mathematics required for quantum theory was ready and waiting. Group theory, the cornerstone of much of theoretical physics of the last fifty years, had its origins in fundamental mathematics of the 18th and 19th centuries. And, when it comes to Superstrings, a topic at the frontiers of contemporary theoretical physics, the mathematical tools of cohomology and differential geometry are waiting to be used. On the face of it, this apparently perfect marriage between abstract mathematics and the study of the physical world is as improbable as discovering that a piece of modern sculpture fits exactly as the missing component of some complex new engine!

How is it possible to account for this unreasonable effectiveness of mathematics and for the powerful role it plays in physics today? One approach is to take the hint offered by Galileo and view mathematics as a language. Just as natural language is used for everyday thought and communication, so too, physics has to make use of whatever mathematical languages happen to be lying around. Mathematics, in this view, is a tool and, like the hammer or screwdriver, we select the available tool that best fits the job.

3. Mathematics as Language

It is common to talk of "the language of mathematics". But is mathematics really a language? Does it possess the various properties that are characteristic of other natural languages? Clearly mathematics does not have the same fluency as a natural language and, even more obviously, it is rarely spoken aloud. This suggest that mathematics is really a more restrictive limited form of language. Nevertheless, the suggestion is that everything mathematics can do must ultimately find its origin in language. This means that the rich and abstract proofs and theorems of mathematics can ultimately be traced back to thoughts and arguments that were once voiced in language--albeit in a long winded and cumbersome way. Now, it is obvious that mathematics doesn't look anything like natural language. Mathematics deals with numbers and symbols, it is used to make calculations and it's form is highly abstract. On the other hand, all these features may already be enfolded within natural language. The power of language lies in the way meaning can be conveyed through form and transformation. The Ancient Greeks, for example, realized that truth could be arrived at through various patterns of sentences.

All men are mortal
Socrates is a man

Therefore: Socrates is mortal.

Or, to take another pattern,

Some mathematicians are clever.
All mathematicians are animals.
Therefore: Some animals are clever.

What is striking about these patterns is that the truth of the conclusion does not depend on the content of the sentences but on their form. In other words, substitutions do not affect the validity of the proof:

All [cats] are [wanderers].
[Minou] is a [cat].
Therefore: [Minou] is a [wanderer].

Clearly these patterns and substitutions have something in common with algebra. Other transformations are also possible within language.

From:

John shut the door

we get:

The door was shut by John.

These are only a few of the great range of abstract operations possible within language. Indeed the linguist Noam Chomsky3 has argued that this ability arises genetically and is inherent in all human thought. To take Chomsky's idea even further we could say that mathematics has isolated and refined several of the abstract elements that are essential to all human languages. An extreme form of this argument would be to say that while mathematicians may make abstract discoveries and develop new mathematical forms, in the last analysis they are simply representing something that is inherent in human thought and language.

The normal way we express and communicate our thought is through language and mathematics becomes an formal extension of this process. So when physicists seek a rational language in which to express their insights, they simply take what happens to be at hand - the best available mathematics. It is not therefore surprising that mathematics happens to work.

Mathematics has played a vital role in raising the speculations an earlier age to the highest peaks of intellectual enquiry. But I am now putting forward the hypothesis that physicists have, in fact, no alternative. Mathematics has been forced on them as the only language of communication which can also serve to make, with precision and economy, quantitative predictions and comparisons. And, when no Isaac Newton happens to be around to develop a new mathematical language hand in hand with new physical insights, then physics has to make do with what is available.

In those cases in which the form of the mathematical language makes a perfect marriage with to the content of the physical ideas, then the communication and development of physics is highly successful. But this may not always be the case. Sometimes it may turn out that a particular mathematical language is forced, by physics, to say things in cumbersome ways. The mathematics actually gets in the way of further creativity. At the other extreme, it is the very ease of expression that drives a theory in a particular direction so that mathematics actually directs the evolution of physics, even when new physical insights are lacking. In other words, I want to question Wigner's claim that mathematics is unreasonably effective. For it could be that the whole thing is an illusion brought about because physics has no other language in which to communicate quantitative statements about the world. In the past decades there has been much talk about paradigm shifts and scientific revolutions - yet it is still possible to retain the same mathematical language after such a radical shift. In short, the whole baggage of unexamined presuppositions that are inherent in the mathematics are carried over to the new physics.

Any writer knows that language has the power to take over his or her ideas. Words have their own magic, and a style, once adopted, will gather its own momentum. It has been said that a writer is possessed by all the texts that have been previously written. As soon as we put pen to paper and chose a particular literary form then what we write is, to some extent predetermined. I would suggest that the same is true of physics. That the adoption of a particular mathematical language will subtly direct the development of new ideas. Moreover there are times when mathematics may actually block the operation of a free, creative imagination in physics. Since mathematics occupies such a prominent place in physics today, these are vital questions to be explored.

In arguing that mathematical languages direct and influence our thought in science, we now see that the real danger arises from always focusing on the physical ideas and not giving attention to the language in which they are expressed! As long as physicists view mathematics simply as a tool then it is possible to ignore the subtle but very powerful influence it has over the way they think and how they express their thoughts. In fact, I believe that a good argument can be made that a particular form of mathematics has been blocking progress in physics for decades- this is the Cartesian co-ordinate system, a mathematical form that has survived several scientific revolutions!

A major problem facing modern physics is that of unifying quantum theory with relativity. One theory deals with discrete, quantized processes below the level of the atom. The other with the properties of a continuous space-time. While it is certainly true that deep physical issues must be resolved before significant progress can be made, I would also argue that the mathematical language in which the quantum theory is expressed is at odds with what the theory is actually saying. While quantum mechanics and quantum field theory are a truly revolutionary approaches, the mathematics they are based on goes right back to Descartes--to the same Cartesian co-ordinates we all learned at school. For three hundred years physics has employed the language of co-ordinates to discuss the movement of objects in space and time. Later developments like the calculus also rely upon this idea that space can be represented by a grid of co-ordinates. But it is this same mathematical language that is at odds with the revolutionary insights of quantum theory. Cartesian co-ordinates imply continuity, and the notion of space as a backdrop against which objects move. So whatever new insights physics may have in this area, they are still being expressed in an inappropriate language. This, I believe, represents a major block to thinking about space and quantum processes in radically new ways.

The example of how the Cartesian grid has dominated physics is rather obvious. But there may be many other, and more subtle, ways in which particular mathematical forms are currently directing science and limiting the possibilities for its development.

4. Mathematics Beyond Language

But is it really true that mathematics is nothing more than a limited and abstract version of natural language? I would argue that mathematics is both more, and less, than a language. Since it involves highly codified forms, mathematics makes it easy to carry out calculations, to demonstrate proofs and to arrive at true assertions. But, in my opinion, this is only a surface difference, a feature of the convenience and economy of mathematics over ordinary language. A more significant way in which mathematics goes beyond language is that it involves a particular kind of visual and sensory motor thinking that does not seem to be characteristic of ordinary language. Some parts of mathematics deal with the properties and relationships of shapes. While these properties can be generalized to many dimensions and to highly abstract relationships, nevertheless, mathematicians have told me that their thinking in these particular fields enters regions which do not involve language in any way. It calls upon a sort of direct, internal visualization and may even involve an internal sense of movement and of tiny muscular reactions. This "non-verbal" thinking may also take place in other fields of mathematics and appears to involve a form of mental activity that goes beyond anything in the domain of a spoken or written language. It could be that, at such times, mathematical thought has direct access to a form of thinking that is deeper and more primitive than anything available in any natural language. This pre-linguistic mental activity may be the common source from which both mathematics and ordinary language emerge.

On the other hand, mathematics is also less than a language, in that it lacks the richness, the ability to deal with nuance, the inherent ambiguity and the rich strategies for dealing with this ambiguity. In this sense, mathematics is a limited, technical language in which much that is of deep human value cannot be expressed.

5. Mathematics and Music

It is possible to explore the nature of mathematics, and its relationship to physics, in another direction. By comparing it to music. Mathematics is an abstract system of ordered and structured thought, existing for its own sake. It is possible to apply a similar description to music. Indeed the 20th century composer, Edgar Varese, has written that "music is the corporealization of thought". Listening to Bach, for example, is to experience directly the ordered unfolding of a great mind. This suggests that music and mathematics could be related in some essential way. On the other hand who would employ music to express a new theory of the universe? (But could this simply be a prejudice that is characteristic of our earth-bound consciousness? Do beings in some remote corner of the universe explore the nature of the universe in music and art?)

Music and mathematics are similar, yet different. Indeed, I believe that both the strengths and the weakness of mathematics lie in this difference. Mathematics has developed to deal with proof and logical truth in a precise and economical way. Mathematics also makes a direct correspondence with the physical world through number, calculations and quantitative predictions.

While it could be said that music is "true" in some poetical sense and that the development of a fugue has a logical ordering that is similar to that of a mathematical proof, on the other hand these are not the primary goals of music. Music deals with the orders of rational thought, yet it is also concerned with the exploration of tension and resolution, with anticipation, with the control of complex sensations of sound and with the evolution and contrast of orders emotion and feeling. To borrow a Jungian term, music could be said to be more complete, for it seeks a harmony between the four basic human functions; thought balanced by feeling and intuition by sensation. While mathematicians may experience deep emotions when working on a fundamental piece of mathematics, unlike composers, their study, per se, is not really concerned with the rational ordering of these emotions or with the relationships between them. The greatest music, however, moves us in a deep way and leaves us feeling whole. It engages thought and emotion, it expresses itself through the physical sensation of sound.

In this sense it could be said that physics, with its reliance on the language of mathematics, must always present an incomplete picture of the universe. Its language is impoverished, for it lacks this basic integration of the four human functions. It can never fully express the essential fact of our confrontation with, participation in, and understanding of nature.

But is it possible, in wonder, that, in the distant future, science, inspired by the example of music, may develop a more integrated and versatile language, one which would have room, perhaps, for the order of emotion and direct sensation while, at the same time, retaining all the power of a more conventional mathematics?.

There is yet another significant way in which "the language of music", and of the other art, differs from mathematics. While all these languages are concerned with relationships and rational orders of thought, the arts are able to unfold these orders in a more dynamical way by exploring the way order is generated in the act of perception itself. Quantum theory is also concerned with the indivisible link between the observer and the observed. And this suggests that it would be to the advantage of physics to develop a similar flexibility in its basic language giving it the ability to explore the rich orders that lie between the observer and the observed.

Let me explain what I mean. A great work of art possesses a rich internal order. In music, for example, a theme may be transposed, inverted, played backwards and otherwise transformed in a variety of ways which still retain a certain element of its order. Of course, this is only one simple example of the sorts of order explored in a musical composition, indeed the order of great music is so rich as to defy complete analysis. Likewise, a painting contains complex relationships between its lines, masses, areas, colors, movements and so on. In some cases such objective orders may have much in common with the sorts of order that are found in mathematics. But what makes any work of art come alive is its contemplation by the human observer:- Music played in a vacuum is not music, art that is never seen is not art. For the work of art arises in that dynamic interaction between the active perception, intelligence, knowledge and feeling of the viewer and the work itself.

To take a particular example, some of the drawings of an artist like Rembrant, Picasso or Mattise or a Japanese master appear, on the surface, to be extraordinarily simple. Few marks appear on the paper when contrasted with, for example, the detailed rendering done by an art student. A trivial analysis would suggest that the sketch contains "less information" than the detailed rendering and that its order is relatively impoverished. Yet the confrontation of a viewer with a Mattise drawing is a far richer experience in which complex orders of thought and perception are evoked. To make the slightest change in position, direction, gesture or even thickness of a single line can destroy the balance and value of a great drawing, but may have only a negligible effect on a student work. In this sense great art has an order of such richness, subtlety and complexity that it is beyond anything that can be addressed in current mathematics. Yet it is something to which the trained viewer can immediately respond.

Indeed, the rich order of the drawing lies not so much in some objective order of the surface marks on the paper, but in the whole act of perception itself and in the way in which the drawing generates a hierarchy of orders within the mind. Lines evoke anticipations in the mind that may be fulfilled in harmonious or in unexpected ways. The mind is constantly filling in, completing, creating endless complex orders. A single line may suggest the boundary of a shadow, the outline of a back or it may complete a rhythm created by other lines. Indeed the act of viewing a drawing could be said to evoke an echo, or resonance, of the whole generative process by which the drawing itself was originally made. The essence of the drawing does not therefore lie in a static, objective order--the sort of thing that can be the subject of a crude computer analysis involving the position and direction of a number of lines. Rather, it is a rich dynamical order, an order of generation within the mind. Through his or her art, the creator of the drawing has called upon the nature of the subject, the history of art, and on all the strategies that are employed in perception. So standing before a drawing involve a deep and complex interplay between the work itself, the visual center of the brain, memory, experience, knowledge of other paintings, and of the human form. The eyes, memory, mind and even the body's sensory-motor system become involved in the generation of a highly complex order, an order in which every nuance of the drawing has its part.

The order within an economical drawing may, therefore, be far richer than we first suspect. For its power lies not so much in some surface pattern of the lines but in the controlled and predetermined way in which these lines generate, through the act of perception itself, infinite orders within the mind and body. While attention has certainly been given, by researchers in Artificial Intelligence, to what is called the early processes of vision, it is clear that the sort of order I am talking about lies far beyond anything that mathematics or artificial intelligence could analyze or even attempt to deal with at present.

I feel that the description of complex orders of perception and generation is a rich and powerful area into which mathematics should expand. It may also have an important role to play in physics. Quantum theory, for example, is concerned with the indissoluble link between observer and observed and it would be interesting to make use of a mathematics which can express the infinite orders that are inherent in this notion of wholeness.

A similar sort of argument applies to music. Some musicologists have gone so far as to analyze music by computer, and to calculate its "information content ", concluding, for example, that "modern music" contains more information than baroque music! But the essence of music does not lie in some measure of its objective information content but in the rich and subtle activity it evokes within the mind. Music and art are seeds that, in a controlled and deliberate way, generate a flowering of order and meaning within the mind and body of the listener.

To return to an earlier point; this generative order suggests a reason why great music could indeed act as a metaphor for a theory of the universe. Music is concerned with the creation and ordering of a cosmos of thought, feeling, intuition and sensation and with the infinite dynamical orders that are present within this cosmos. In this sense, music could be said to echo the generation and evolution of a universe. Clearly our present mathematics lacks this essential dimension. But could, in fact, mathematics move in such a direction? A new mathematics would not simply offer a crystallization of thought but also explore the actual generative activity of the orders of this thought within the body and mind. Such a new formal language would represent a deep marriage between mathematics and the arts. It would involve a mathematics that requires the existence of another mind to complete it, in an ordered and controlled way, and, in so doing, this mathematics would becomes the germ of some, much deeper order.

6. Mathematics and the Brain

Let us return again to the question of the unreasonable effectiveness of mathematics. As we have seen, one answer is to consider mathematics as a language, indeed the only available language that can deal, in an economical and precise way, with quantitative deductions about the world. Mathematics, in this sense, is a restricted form of natural language. But, in other ways, it goes beyond language. Physics, however, is always in the position of being forced to use mathematics to communicate at the formal level. The question, therefore, is not so much one of the unreasonable effectiveness of mathematics, but of physicists having no real alternatives.

But there may be other ways of looking at this question. One way is to suggest that mathematics, in its orders and relationships, is a reflection of the internal structure and processes of the brain. In moving towards the foundations of mathematics one would therefore be approaching some sort of direct expression of the controlling activities of the brain itself. And, since the brain is a physical organ that has evolved through its interactions with the material world, it is inevitable that the brain's underlying processes should model that world in a relatively successful way. Human consciousness has developed, in part, as an expression of our particular size and scale within the environment of our planet. It is a function of the particular ranges of senses our bodies employ, and of our need to anticipate, plan ahead, hold onto the image of a goal and remember. Moreover consciousness has created, and been formed by, society and the need to communicate. It has brought us to the point where we can ask, for example, if we think because we have language or, if we have language because we think? Or if the answer could lie somewhere in between.

According to this general argument, the brain's function is a direct consequence of, and a reflection of, our particular status as physical and social beings on this planet. Mathematics, moreover, is a symbolic expression of certain of the ordered operations of this brain. It should come as no surprise, therefore, that mathematics should serve as a suitable language in which to express the theoretical models that have been created by this same brain.

This whole question of the formal strategies employed by the brain is the province of cognitive psychology. One of the pioneers in that field was Jean Piaget5. Piaget's particular approach was to suggest that the basis of our thought and action could be traced to the logic of the various physical transactions we had with the world during our first weeks, months and years. Piaget believed that these same logical operations are also present in mathematics and, in this respect, he had a very interesting point to make. It is well known, he pointed out, that mathematics can be arranged in a hierarchical structure of greater and greater depth. In the case of geometry, for example, the top, and most superficial, level is occupied by those semi-empirical rules for surveying and calculating shapes that were known to the Egyptians and Babylonians. Below that could be placed the more fundamental, axiomatic methods of the ancient Greeks. The history of geometry demonstrates the discovery of deeper and more general levels, Euclidian geometry gives way to non-Euclidian, beneath geometry is topology, and topology itself is founded on even more general and beautiful mathematics. The longer a particular topic has been studied, the deeper mathematicians are able to move towards its foundations.

But Piaget, pointed out, this historical evolution is a direct reversal of the actual development of concepts of space in the infant. To the young child, the distinction between intersecting and non-intersecting figures is more immediate than between, say, a triangle, square and circle. To the infant's developing mind, topology comes before geometry. In general, deeper and more fundamental logical operations are developed earlier than more specific rules and applications. The history of mathematics, which is generally taken as a process of moving towards deeper and more general levels of thought, could also be thought of as a process of excavation which attempts to uncover the earliest operations of thought in infancy. According to this argument, the very first operations exist at a pre-conscious level so that the more fundamental a logical operation happens to be, the earlier it was developed by the infant and the deeper it has become buried in the mind. Again, this suggests a reason why mathematics is so unreasonably effective, for the deeper it goes the more it becomes a formal expression of the ways in which with interact with, and learn about, the world.

But, it could be objected, if the history of mathematics and, to some extent, of theoretical physics, is simply that of uncovering, and formalizing, what we already know then how is it possible to create new ideas, like Einstein's relativity, that totally lie outside our experience? The point is, however, that this equality or interdependence of space and time was already present in all the world's language. Rather than coming to the revelation that time and space must be unified then have never really been linguistically separated! According to this general idea, what may appear to be novel in physics and mathematics is essentially the explicit unfolding of something that is already implicit within the structuring of human thought--of course physics itself also makes use of empirical observations and predictions. For this reason, the intelligent use of mathematics as a language for physics will necessarily make sense.

Piaget's notion, that the evolution of mathematics and physics is forever reaching towards the deepest structures of the mind, is certainly interesting. However, I feel that there is a certain limitation in the approach of cognitive psychology, with its emphasis upon strategies and programs of the brain, on successions of logical steps and on algorithms of thought. There is not sufficient space in this article to develop any detailed arguments, but I believe that, while cognitive psychology may produce some valuable insights, in its present form it does not capture the true nature of human intelligence in general, and mathematics in particular. Formal logic is an impoverished way of describing human thought and the practice of mathematics goes far beyond a set of algorithmic rules. The mathematician Roger Penrose7 has, for example, produced compelling arguments why machine intelligence must be limited--a Turing machine, or indeed any other algorithmic device, will never be able to carry out all the sorts of things that a human mathematician can do. Mathematics may indeed reflect the operations of the brain, but both brain and mind are far richer in their nature than is suggested by any structure of algorithms and logical operations.

7. Mathematics and Archetypes

In this final section I am going to become more speculative and explore yet another approach to the question of the unreasonable effectiveness of mathematics. I want to suggest that mind and matter, brain and consciousness are two sides of a single process, something that emerges out of a deeper and hitherto unexplored ground. In this sense the order of generation that gives rise to the universe has a common source with the generative order of consciousness. In its deepest operation, therefore, our intelligence could be said to mirror the world. But what can one say about the nature of this source? According to the classical Chinese philosopher, Lau Tzu, "the Tao which has a name in not the Tao", which seems to say it all.

Of course, the idea of an unknown, unconditioned source which is the origin of matter and consciousness may seem far fetched to many readers. But it is, after all, simply another way of accounting for the unreasonable effectiveness of mathematics. Our own age is out of sympathy with such sweeping assertions as "God is a mathematician", but suppose one suggests that mind and the universe have an common order and that the source of material and mental existence lies in a sort of unconditioned creativity, and in the generation of orders of infinite subtlety and complexity8? While the nature of such an order may never be explicitly known in its entirely, it may still be possible to unfold certain of its aspects through music, art and mathematics. The great aesthetic joy of mathematics is not, therefore, far from the joy of music or any great art, for it arises in that sense of contact with something much greater than ourselves, with the heart of the universe itself. Mathematics is effective when it becomes a hymn to this underlying order of consciousness and the universe, and when it expresses something of the truth inherent in nature.

This idea has been expressed in other ways. Carl Jung, for example, spoke of the archetypes. This is a difficult concept to convey in a short definition but, very roughly, the archetypes could be taken as those dynamical orders, unknowable in themselves, that underlie the structure of the collective unconscious. The archetypes are never seen directly but their power can be experienced in certain universal symbols. In his more speculative moments, Jung also hinted at something that lay beyond matter and mind, but included both. This psychoid, as he called it, is related to the archetypes and suggests that the same underlying ordering principles give birth and structure to both matter and mind. Just as human consciousness arises out of the collective unconscious, so too the universe itself arises out of something more primitive. Again we meet this notion that the same underlying order gives rise to both matter and mind.

Of particular interest is the importance that Jung placed upon numbers. Numbers, according to Jung, are direct manifestations of the archetypes and must therefore be echoes of the basic structuring processes of the universe itself. It is certainly true that numbers are mysterious things. To return, for a moment, to the connection between mathematics and language. When it comes to language, it is a basic axiom of linguistics that "the sign is arbitrary". In other words, the meaning of a world does not lie in how it sounds or the way it is written but in the way it is used. If you want to know the meaning, the philosopher Wittgenstein said, look for the use. By contrast, the basic units of mathematics, the numbers, are totally different, they are not arbitrary but have a meaning and existence of their own. While the names given to the numbers may be arbitrary, the numbers themselves are not, 0, 1, 2, 3, are not symbols whose meaning changes with time and use but are the givens of mathematics. In a sense they are almost platonic. It has been said, for example, that God made the numbers and the rest of mathematics is the creation of human intelligence. It is these same numbers that, Jung claims, are manifestations of the archetypes. Indeed Jung's argument does have a ring of truth about it for numbers are certainly curious things and the unfolding of their properties remains one of the most basic forms of mathematics. Could it be true, as the Jungians suggest, that the numbers are expressions of the archetypes or orders that underlie the universe and human consciousness?6

Curiously enough, this idea may have found favour with one famous mathematician. One of the most brilliant pure mathematicians in this century, S. Ramanujan, gave little value to mathematical proof but appeared to arrive at his remarkable theorems in number theory by pure intuition alone. Ramanujan himself, however, believed that these profound results were given to him by a female deity. In Jung's terminology, this deity would also be a manifestation of the archetypes.

So, to Ramanujan, the whole order of mathematics, with its underlying truth and beauty, essentially lies in a domain beyond logical truth and rational argument. It is something which can, at times, be touched directly by the mathematician's intuition and in a way that appears almost sacred. As to the nature of this domain, we can call it the archetypes, psychoid, ground of being or unconditioned, creative source. But what does it matter? What counts is that a remarkable mathematician bypassed rational argument and the need for vigorous proof and picked out outstanding theorems out of the air. And what is equally staggering is that, in all likelihood, these symphonies of pure thought may one day have totally practical applications in the real word.

8. Conclusion

The unreasonable effectiveness of mathematics remains an open question, although I have given some suggestions as to why it appears to work. I have also argued that mathematics may not always be as effective as we suppose, for physical ideas are sometimes forced to fit a particular mathematical language, in other cases the very facility of the language itself may drive physics forward, irrespective of any new physical ideas!

I have also suggested ways in which improvements in the formal language of physics could be advanced. A major area would be to discover a mathematics of complex and subtle orders, a formal way of describing what seems, to me, to be an essential feature of the universe. There have recently been several attempts to describe complex orders--Mandelbrot's fractal theory is capable to describing and generating figures of infinite complexity; David Bohm's notion of the implicate order is a powerful concept but has yet to find an appropriate mathematical expression.9

Finally, I have also argued that there are times when the mathematical language of physics fails to capture the essential fact of our being in the universe. And here I must reveal another prejudice. Physics, to me, has always been concerned with understanding the nature of the universe we live in; a way of celebrating and coming to terms with our existence in the material world, rather than a matter of discovering new technologies and accumulating more knowledge. In is in this light that I have criticized the role of mathematics in physics and have hinted at the way new language forms could be developed. Of course I acknowledge the great service that mathematics has done for physics, how it has lifted it from speculation to precision, and, of course, I recognize the great power and beauty of mathematics that is practiced for its own sake. But here, at the end of the 20th century we must not rest on our laurels, the whole aim of our enterprise is to penetrate ever deeper, to move towards a more fundamental understanding and a more complete celebration of the universe itself. In this undertaking in which prediction, calculation and control over the physical world also have a place but they do not become the whole goal of the scientific enterprise. It is for this reason that I am urging physicists to play closer attention to the mathematical language they use every day.

This whole concern with discovering and portraying the complex orders of nature, was also a preoccupation of the writer Virginia Woolf. Virginia Woolf was concerned with the order of the moment, with crystalizing, in language, the complex sensations, experiences and memories that make up each instant in a persons life. She recognized that, in the last analysis, the success of this enterprise depends on creating a fitting means of expression, on language and on words. Her own observations on this process convey precisely what I have been attempting to say in this essay

"Life is not a series of gig-lamps symmetrically arranged; but a luminous halo, a semi-transparent envelope surrounding us from the beginning of consciousness to the end. Is it not the task to the novelist to convey this varying, this unknown and uncircumscribed spirit, whatever aberration or complexity it may display, with as little mixture of the alien as possible?"

For James Joyce it is the epiphanies or transcendent moments of life that have a special richness. They can occur at any instant and it is the business of language to capture these , even "transmuting the daily bread of experience into the radiant body of evolving life". For Virginia Woolf this radiant force of the moment must be captured by language "it is or will become a revelation of some order; is a token of some real thing behind appearances; and I make it real by putting it into words.

9. References

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4. For a preliminary discussion on the role of language in science see, A.J.Ford and F. D. Peat, Foundations of Physics 18, 1233-1242, 1988.

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6. M-L von Franz, Number and Time, Northwestern Universty

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7. R. Penrose, " Question Physics and conscious thought" in Quantum Implications: Essays in Honour of David Bohm, eds. B.J.Hiley and F. David Peat., Routledge and KeganPaul, London and New York, 1987.

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8. See, for example, F. David Peat, Synchronicity: the Bridge Between Matter and Mind, Bantam, N.Y., 1987.

9. A discussion of complex orders is given in, D. Bohm and F.D. Peat, Science, Order and Creativity, Bantam, N.Y., 1987.